Level statistics inside the core of a superconductive vortex

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Microscopic theory of the type of Efetov's supermatrix sigma-model is constructed for the low-lying electron states in a mixed superconductive-normal system with disorder. The developed technique is used for the study of the localized states in the core of a vortex in a moderately clean superconductor ($1/\Delta \ll \tau \ll \omega_0^{-1} = E_F/\Delta^2$). At sufficiently low energies $\epsilon \ll \omega_{Th}$, the energy level statistics is described by the "zero-dimensional" limit of this supermatrix theory, with the effective "Thouless energy" $\omega_{Th} \sim (\omega_0/\tau)^{1/2}$. Within this energy range the result for the density of states is equivalent to that obtained within Altland-Zirnbauer random matrix model of class C. Nonzero modes of the sigma-model increase the mean interlevel distance ω_0 by the relative amount of the order of $[2\ln(1/\omega_0\tau)]^{-1}$.

There is a great deal of activity during the last 1.5 decades directed to the study of electron energy levels and wavefunctions in disordered normal metals [1], where they govern low-temperature transport properties. In (s-wave) superconductors, disorder is usually of less importance, since the excitation spectrum is gapful and single-electron states are almost empty at $T \ll \Delta$. The situation is quite different in mixed superconductive-normal systems (for recent reviews see [2]) where the gap in the excitation spectrum can be: i) very low compared to the bulk Δ, or ii) just zero. The example of the first case is presented by an S-N-S sandwich with the thickness L_N of the N region much longer than the superconductive coherence length ξ and the mean free path l. Then at generic values of the phase difference φ between superconductors the gap in the electron spectrum in the N region is of the order of the Thouless energy $E_{Th} = D/L_N^2 \ll \Delta$. To calculate the density of states (DoS) fluctuations at $\epsilon > E_{Th}$, and other mesoscopic effects in such systems, Altland, Simons and Taras-Semchuk developed recently [3] a field theory which is a version of Efetov's supersymmetric matrix sigma-model explicitly taking into account the superconductive coherence induced in the N metal due to the proximity effect. At low enough energies the effective theory they derived is equivalent to the Orthogonal random matrix ensemble at $\varphi = 0$, and to the Unitary ensemble at $\varphi \gg \delta/E_{Th}$ (where δ is the level spacing in the N). Like in the usual normal-metal case [1], mesoscopic fluctuations reveal themselves in the quantities containing products of *both* retarded and advanced Green functions, whereas single, say, retarded Green function is determined by the standard equations of the quasiclassical theory of superconductivity.

Qualitatively different theoretical problem comes about in the second case, ii), mentioned above, which is realized, e.g. in the same S-N-S sandwich at $\varphi = \pi$ [4], or in a variety of situations where the phase of the order parameter rotates due to the presence of an external magnetic field. Now the DoS is non-zero at arbitrary low energies, and quantum interference due to Andreev scattering strongly affects even the average DoS $\langle \rho(\epsilon) \rangle$ at $\epsilon \sim \delta$. General approach to this kind of systems, characterized by the zero *averaged over the whole system* value of the superconductive order parameter, was initiated by Altland and Zirnbauer (AZ) [5], who employed a generalized random-matrix approach. They have shown that the particle-hole symmetry of the Bogolyubov-De Gennes (BdG) Hamiltonian leads to important constraints to be imposed on the random-matrix Hamiltonians. Precise form of the constraint depends on the presence or absence of: a) time inversion symmetry, and b) spin rotation symmetry. Thus AZ identified 4 additional (with respect to the standard Wigner-Dyson theory) classes of random-matrix ensembles appropriate for the description of mesoscopic fluctuations in this kind of S-N-S systems. Crossover between such classes has been considered in [6] using the *space-independent* supermatrix sigma-model. While the AZ approach is certainly highly suggestive, it has the same general limitation as any ad hoc random matrix theory, i.e. the limits of its applicability to some real physical system are left undetermined. Therefore we consider it highly desirable to develop a fully microscopic field theory for the mesoscopic fluctuations in S-N-S systems without a spectral gap.

In the present Letter we develop a microscopic field-theory approach to the generic example of the ii) type of systems, namely, to the core of a superconductive vortex. It is known since Caroli, De Gennes and Matricon (CdGM) [7] that the BdG equations for the electron states near the Abrikosov vortex possess localized solutions with energies well below the bulk Δ. The spacing between these localized levels, ω_0, is of the order of $\Delta/(k_F\xi)$ and disappears in the quasiclassical limit $k_F\xi \to \infty$. Thus it was tempting to consider the vortex core as a kind of a "normal tube" inside a superconductor [8], and, indeed, in many cases such a simplified picture was found [9] to be at least qualitatively correct. Later it was demonstrated [10] that the presence of a quasi-continuum spectrum branch localized on the

vortex follows from general topological arguments; actually, the number of such "chiral" branches coincides with the topological charge of the vortex. However, it is not always possible to consider the chiral branch as a continuos one, as it stands in the quasiclassical calculations. It was shown recently [11,12], that the discreteness of the localized energy levels becomes of real importance in layered superconductors at sufficiently low temperatures. In the previous paper [11] we employed the AZ phenomenological approach to find low-current nonlinearities in the current-voltage relation in a mixed state of a moderately clean superconductor (the mean free path $l \gg \xi$, but $l \ll \xi(k_F\xi)$). In such a case the inverse elastic scattering time $1/\tau$ is much larger than interlevel spacing ω_0, therefore the applicability of an appropriate random-matrix model (which is, in fact, class C of the AZ classification) seems quite natural. Vortex-induced dissipation in another limiting case of a super-clean superconductor with extremely low concentration of impurities ($l \gg k_F\xi^2$) was considered recently by Larkin and co-workers [12]. Here the system of electron levels in the core was found to be extremely strongly correlated and almost integrable, with the properties qualitatively different from the moderately clean case [11]. In the present Letter we again consider a moderately clean limit $\omega_0 \ll 1/\tau \ll \Delta$, now within microscopic approach starting from the BdG equations in the presence of impurities-induced Gaussian random potential. We derive the conditions under which the AZ class C statistics is indeed realized in the vortex core, and estimate the scale of non-universal corrections to it. Throughout all the paper we consider purely 2-dimensional superconductor, which is a good approximation for the case of sufficiently strong layered anisotropy; more quantitative conditions can be found in [11].

Below we present a brief description of our method and results, whereas their detailed presentation is postponed to a forecoming paper [13]. It was already mentioned above that in the present problem even the calculation of the average single-particle quantities is not trivial and cannot be done within the quasiclassical theory, as long as low energies comparable to the level spacing ω_0 are considered. Thus our goal here is to derive a field-theory technique for the calculation of the average DoS $\langle\rho(\epsilon)\rangle = \langle\sum_j \delta(\epsilon-\epsilon_j)\rangle$. To average the Green function over disorder, we use a standard trick [1] of representing it as the functional integral over both Grassmann (χ) and usual complex (S) fields which combine into the superfield Φ. The most direct way would be to work with real-space-dependent superfield $\Phi(\mathbf{r})$ corresponding to the standard representation of the BdG Hamiltonian. In this way we would obtain a field theory in terms of $Q(\mathbf{r})$ supermatrix, depending on two spatial coordinates r_x, r_y. On the other hand, low-lying states of the chiral branch depend upon a single quantum number only (in the absence of disorder it is just the angular momentum), as well as for a generic 1D problem. Therefore, in the basis of such states the BdG Hamiltonian can be represented as a random $N\times N$ Hermitean matrix (where $N \sim \Delta/\omega_0$ is the total number of the localized states in the core) of the certain structure and symmetry which we will discuss below. In the clean limit, $1/\tau \ll \Delta$, the admixture of delocalized ($\epsilon > \Delta$) states to the low-lying ones can be neglected. Thus it is convenient first to reduce the full 2D problem to a sort of random matrix problem that can be further reduced to the 1D field theory, explicitly containing the chiral spectrum branch only.

In the basis of the chiral CdGM states $\Psi_\mu(\mathbf{r}) = A(J_{\mu-1/2}(k_Fr), J_{\mu+1/2}(k_Fr))^T e^{i\mu\theta}e^{-K(r)}$ determined in [7] (here $A \sim \sqrt{k_F/\xi}$ is the normalization constant, θ is the azimuthal angle in the real space, $\mu \in [-N/2, N/2]$ is the angular momentum that takes half-integer values, and $K(r) = (1/\hbar v_F)\int_0^r \Delta(r')dr'$), the full Hamiltonian takes the form $\langle\mu|\hat{H}|\mu'\rangle = \omega_0\mu\delta_{\mu,\mu'} + \langle\mu|\hat{V}|\mu'\rangle$ where the second term is due to the random white-noise impurity potential $U(\mathbf{r})$ with the variance $\langle U(\mathbf{r})U(\mathbf{r}')\rangle = \delta(\mathbf{r}-\mathbf{r}')/(2\pi\nu\tau)$; correspondingly, in the functional integral one should use μ-dependent supervector Φ_μ instead of the superfield $\Phi(\mathbf{r})$. This Hamiltonian obeys the symmetry

$$\hat{H} = -\hat{\gamma}\hat{H}^T\hat{\gamma}^T; \quad \langle\mu|\hat{\gamma}|\mu'\rangle = (-1)^{\mu+1/2}\delta_{\mu+\mu'}, \tag{1}$$

which follows from an identity $\Psi_{-\mu}(\mathbf{r}) = (-1)^{\mu+\frac{1}{2}}i\tau_y\Psi^*_\mu(\mathbf{r})$ that reflects the basic symmetry property of the BdG Hamiltonian. Here we introduced the Pauli matrices $\tau_{x,y,z}$ in the 2×2 Nambu space.

The standard way to solve a complicated random matrix problem is to represent it in a form of the effective field theory. In order to reduce the random matrix problem given by Eq. (1) to the 1D field theory we make a continuous Fourier transform (considering N as very large, which is possible since the energy range $\epsilon \ll \Delta$ is studied below) from the momentum variable μ to the "angle" $\phi \in [0, 2\pi)$, so our superfield will be defined as $\Phi(\phi) = \sum_\mu \Phi_\mu e^{-i(\mu-1/2)\phi}$. Now we can write down an expression for the 'partition function' ($\epsilon_+ \equiv \epsilon + i\delta$):

$$Z^R(\epsilon) = \int \exp i\left\{\int \frac{d\phi}{2\pi}\Phi^*(\phi)\left(\epsilon_+ - i\omega_0\frac{\partial}{\partial\phi} - \frac{\omega_0}{2}\right)\Phi(\phi) - \iint \frac{d\phi d\phi'}{(2\pi)^2}\Phi^*(\phi)V(\phi,\phi')\Phi(\phi')\right\} D\Phi^*(\phi)D\Phi(\phi). \tag{2}$$

Matrix elements $V(\phi,\phi')$ of the random potential in the ϕ-space obey the symmetry relationship that follows from Eq. (1) and are given by

$$V(\phi,\phi') = -e^{i(\phi-\phi')}V^*(\phi+\pi,\phi'+\pi) = A^2\int d^2\mathbf{r}\, w_{\phi\phi'}(r,\theta)U(\mathbf{r})e^{-2K(r)}. \tag{3}$$

The function $w_{\phi\phi'}$ can be presented, using summation formulae for the Bessel functions, as

$$w_{\phi\phi'}(r,\theta) = \left(1 - e^{i(\phi-\phi')}\right)\,\exp\left\{-ik_F r\left[(\sin\phi - \sin\phi')\cos\theta + (\cos\phi - \cos\phi')\sin\theta\right]\right\}. \tag{4}$$

All features of the theory are encoded in the pair correlator $\mathcal{W}(\phi_1,\phi_2,\phi_3,\phi_4) = \langle V(\phi_1,\phi_2)V(\phi_3,\phi_4)\rangle$ where the averaging is performed over the Gaussian distribution of the impurity's potential $U(\mathbf{r})$. Since the typical value of $k_F r \sim k_F\xi \gg 1$, the correlator $\mathcal{W}(\phi_1,\phi_2,\phi_3,\phi_4)$ is essentially non-zero only when the oscillating exponents in Eq. (4) nearly cancel each other, i.e. when its arguments ϕ_i are nearly pair-wise coinciding [13]:

$$\mathcal{W}(\phi_1,\phi_2,\phi_3,\phi_4) = \frac{g\omega_0^2}{\pi}T(\phi_1-\phi_2+\pi)(2\pi)^2\left[\delta(\phi_1-\phi_4)\delta(\phi_2-\phi_3) - e^{i(\phi_1-\phi_2)}\delta(\phi_1-\phi_3+\pi)\delta(\phi_2-\phi_4+\pi)\right], \tag{5}$$

where $g = 2A^4/\pi\nu\tau\omega_0^2 k_F^2 \sim 1/\omega_0\tau \gg 1$ and the kernel T is given by

$$T(\phi) = \begin{cases} \frac{\pi}{2}\left|\cot\frac{\phi}{2}\right|, & \text{if } |\phi| > \frac{1}{\sqrt{N}}; \\ \sim\sqrt{N}, & \text{if } |\phi| < \frac{1}{\sqrt{N}}. \end{cases} \tag{6}$$

The δ-function approximation (5) for the correlator $\mathcal{W}$ is valid as long as the scale of the angular variations of the field $\Phi(\phi)$ (below it will be seen to be $\ell = [g\ln(N/g)]^{-1}$) is longer than the actual [13] width $w(\phi_1-\phi_2) \sim |N\sin(\phi_1-\phi_2)|^{-1}$ of those δ-functions. Thus the following derivation is strictly valid under the condition $w(\ell) \ll \ell$ which is equivalent to:

$$\tau\sqrt{\omega_0\Delta} \gg \ln\Delta\tau. \tag{7}$$

Note that this condition is stronger than just the clean limit condition $\tau\Delta \gg 1$. In what follows we will assume this condition to be always fulfilled.

The next step of the σ-model derivation is to average the partition function (2) using Eqs. (5), (6). Before doing that we need to take explicitly into account the symmetry (1), (3) which amount to the doubling of the number of components of the supervector $\Phi(\phi)$ (a similar procedure in the standard approach [1] is related with the time-reversal symmetry). Thus we introduce (cf. with a similar procedure in [6]) an additional 2×2 "particle-hole" (PH) space and define a 4-dimensional supervector field $\psi(\phi) = 2^{-1/2}\left(\Phi(\phi), e^{i\phi}\Phi^*(\phi+\pi)\right)^T$. Next we define the bar-conjugated superfield as $\bar\psi(\phi) = \psi^+\sigma_z = [C(\phi)\psi(\phi+\pi)]^T$ with $C(\phi) = -e^{-i\phi}\sigma_z C_0$, where σ_z is the Pauli matrix acting in the PH space, and

$$C_0 = \begin{pmatrix} 0 & 1 \\ k & 0 \end{pmatrix}_{ph}, \qquad k = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{fb}, \tag{8}$$

with FB meaning the Fermi-Bose space. After the averaging over disorder the effective action $\mathcal{A}\{\psi\}$ entering the functional integral for the retarded Green function $\mathcal{G}^R(\epsilon) = -i\int \Phi_a\Phi_a^+ \exp[\mathcal{A}\{\psi\}]\mathcal{D}\psi^*\mathcal{D}\psi$ (where a implies either bosonic or fermionic component) can be written as

$$\mathcal{A}\{\psi\} = i\int\frac{d\phi}{2\pi}\bar\psi(\phi)\left(\epsilon_+\sigma_z - i\omega_0\frac{\partial}{\partial\phi} - \frac{\omega_0}{2}\right)\psi(\phi) - \frac{g\omega_0^2}{\pi}\iint\frac{d\phi_1 d\phi_2}{(2\pi)^2}T(\phi_1-\phi_2+\pi)\bar\psi(\phi_1)\psi(\phi_2)\bar\psi(\phi_2)\psi(\phi_1). \tag{9}$$

The second term in the action (9) is very similar to that of the 1D tight-binding model with off-diagonal random matrix elements with variance decaying as $1/|x|$, as long as we are interested in the scale $|x| \equiv |\phi_1-\phi_2+\pi| \ll \pi$. Therefore, we expect the usual 1D localization to be absent in our problem just because of the long-range nature of the off-diagonal disorder (cf. [14]).

There is also another suggestive way of considering this term, which helps to gain some intuition about its effect. Namely, one can think of the variable ϕ as an angle associated with the 2D quasiparticle momentum $p = k_F\{\cos\phi, \sin\phi\}$. Then the second term in Eq. (9) corresponds to a 2D *particle-hole* scattering which is strongly enchanced in the forward direction. For such a singular scattering one has to define two scattering lengths ℓ and $\ell_{tr} \gg \ell$ (cf. with a similar situation discussed in [15]): $1/\ell \propto g\int d\phi\;\sigma(\phi) = g\ln(N/g)$, and $1/\ell_{tr} \propto g\int d\phi\;\sigma(\phi)\;(1-\cos\phi) = g \gg 1$, where $\sigma(\phi)$ is the differential cross-section and $\phi = \phi_1-\phi_2+\pi$. For the careful evaluation of the logarithmically divergent scattering rate $1/\ell$, one should use the self-consistent Born approximation (SCBA) which takes into account both terms in Eq. (5). It is equivalent to taking into account of the "non-crossing" diagrams that can be generated by a perturbative expansion of $\exp[\mathcal{A}\{\psi\}]$ in powers of g. For

$\epsilon/\omega_0 \ll 1/\ell$, the "crossing diagrams" of the same order in g turn out to be small by the parameter $\ell/\ell_{tr} = 1/\ln(N/g)$. It stands for the usual quasiclassical parameter $(k_F \ell_{tr})^{1-d}$ in this effectively 1D problem.

The existence of the small papameter $1/\ln(N/g) = 1/\ln \Delta\tau$ that allows to neglect the "crossing diagrams", implies that one can derive an effective field theory (nonlinear sigma-model) which describes the low-energy behavior of the averaged Green function $G^R(\epsilon)$ for $\epsilon/\omega_0 \ll 1/\ell = g\ln(N/g)$. This can be done in a standard way [1] by the Hubbard-Stratonovich decoupling of the quartic term in Eq. (9) and a further saddle point approximation controlled by the parameter $1/\ln(N/g)$. Because of the symmetry relationship (1), (3) and the corresponding relationship between $\bar{\psi}$ and ψ, one has to perform both the local decoupling containing $P(\phi)\ \psi(\phi) \otimes \bar{\psi}(\phi)$ and the non-local one containing $R(\phi_1, \phi_2)\ \psi(\phi_1) \otimes \psi^T(\phi_2)$. Under the condition $\ell \gg 1/\sqrt{N}$ given by Eq. (7), both decouplings are important in order to obtain a correct form for the imaginary part of the Green function in a saddle-point approximation $P(\phi) = P_0$, $R(\phi_1, \phi_2) = R_0(\phi_1 - \phi_2)$ which is equivalent to the SCBA:

$$G_\epsilon(\phi) = -\frac{2\pi i}{\omega_0}\sigma_z \theta(-\sigma_z\phi) e^{-|\phi|/\ell} e^{-i\frac{\epsilon}{\omega_0}\phi}, \quad P_0 = \frac{T_0}{\omega_0}\sigma_z, \quad R_0(\phi) = \frac{i}{\pi} T(\phi) G_\epsilon(-\phi), \tag{10}$$

where $\ell^{-1} = g\ln(N/g)$, and $T_0 = \int_0^{2\pi} T(\phi)\frac{d\phi}{2\pi} \approx \frac{1}{2}\ln N$. In general, m-th Fourier harmonics of the kernel $T(\phi)$ is given by $T_m \approx \ln\frac{\sqrt{N}}{|m|}$ for $1 \ll m \ll \sqrt{N}$.

Mesoscopic fluctuations are known [1] to be described by the slow rotations of the saddle-point solution, which are represented in our case as $P(\phi) = U^{-1}(\phi) P_0 U(\phi)$, $\quad R(\phi, \phi') = U^{-1}(\phi) R_0(\phi - \phi') U(\phi')$. The corresponding action that describes the low-energy spectral properties of the CdGM levels, reads:

$$\mathcal{A}_\sigma[Q, U] = -\frac{\pi g}{4} T_0^2 \iint \frac{d\phi_1 d\phi_2}{(2\pi)^2} T^{-1}(\phi_1 - \phi_2 + \pi) \operatorname{Str} Q(\phi_1) Q(\phi_2) - \frac{\pi i}{2}\int \frac{d\phi}{2\pi} \operatorname{Str}\left(\frac{\epsilon}{\omega_0}\sigma_z Q(\phi) - i\sigma_z U(\phi)\frac{\partial U^{-1}(\phi)}{\partial\phi}\right), \tag{11}$$

where $Q(\phi) = U^{-1}(\phi)\sigma_z U(\phi)$, and $U(\phi)$ is a π-periodic, pseudo-unitary ($U^{-1}(\phi) = \bar{U}(\phi)$) matrix. The action (11) is valid for the energies $\epsilon \ll \omega_0/\ell = \tau^{-1}\ln \Delta\tau$.

The supermatrix Q can be represented in the form $Q(\phi) = \sigma_z[1 + W(\phi) + \frac{1}{2}W^2(\phi) + O(W^3)]$ with the supermatrix W being purely off-diagonal in the PH space. Then the symmetry $Q = \bar{Q}$ and convergence arguments lead to the following form for the W_{ph} and W_{hp} blocks:

$$W_{ph}(\phi) = \begin{pmatrix} iz(\phi) & \alpha_1(\phi) \\ \alpha_1(\phi) & 0 \end{pmatrix}_{fb}, \qquad W_{hp}(\phi) = \begin{pmatrix} iz^*(\phi) & \alpha_2(\phi) \\ -\alpha_2(\phi) & 0 \end{pmatrix}_{fb}. \tag{12}$$

Here z is a complex number and α_i are Grassmann numbers. Expanding over $W(\phi)$, we obtain in the quadratic approximation

$$\mathcal{A}_2[W_m] = \frac{\pi}{4}\operatorname{Str}\sum_m \left\{ 2g\left(\sum_{k=0}^{|m|-1}\frac{1}{2k+1}\right) + i\left(m\sigma_z - \frac{\epsilon}{\omega_0}\right)\right\} W_{2m} W_{-2m}, \tag{13}$$

where W_m is the m-th harmonics of the field $W(\phi)$. Note that in Eq. (13) only even harmonics enter; odd harmonics, as well as the "longitudinal" modes, have a larger gap of the order of ω_0/ℓ and are excluded from the sigma-model action.

Eq. (13) sets a characteristic scale L for the angular variations of matrices $U(\phi)$:

$$1/L = g\ln g. \tag{14}$$

This scale should be larger than the scattering length ℓ. Only in this case one can restrict oneself by the lowest term of the gradient expansion in powers of $\partial U/\partial\phi$ that has been used in deriving Eq. (11). Comparing to $\ell = [g\ln(N/g)]^{-1}$ we see that the parameter of the gradient expansion, $\ell/L = \ln g/\ln(N/g)$, is small if the condition (7) is fulfilled.

The length L determines the angular size of the elementary propagator corresponding to the sigma-model (11). In this respect it is analogous to the phase-breaking length for Cooperons (or the system size) in the usual weak-localization problem. The fact that $\ell/L \ll 1$ in our problem tells us that the problem is essentially not ballistic, though it is not diffusive either, since $\ell_{tr}/L = \ln g \gg 1$.

An important property of the action (11) is that it takes a universal form if U is independent of ϕ. It is clear from Eq. (13) that at low energies the main contribution to the functional integral comes from the zero harmonics of $Q(\phi)$,

i.e. the problem reduces to the zero-dimensional supermatrix σ-model. The uniform supermatrix Q is parametrized by 2 real variables (one of which appears to be cyclic) and 2 Grassmann variables, so the final expression for the average DoS is

$$\langle\rho(\epsilon)\rangle = \frac{1}{4\tilde{\omega}_0}\Re\int_0^\pi d\theta\int d\eta d\zeta\frac{\sin\theta}{1-\cos\theta}\left[(1+\cos\theta)+2\eta\zeta(1-\cos\theta)\right]e^{\pi i\frac{\epsilon}{\tilde{\omega}_0}(1-\cos\theta)} = \frac{1}{\tilde{\omega}_0}\left(1-\frac{\sin(2\pi\epsilon/\tilde{\omega}_0)}{2\pi\epsilon/\tilde{\omega}_0}\right). \quad (15)$$

The functional form of this result coincides with the result of the AZ phenomenological approach [5]. However, it is expressed via the renormalized mean level spacing $\tilde{\omega}_0 = \omega_0\left(1+\frac{1}{2\ln g}\right)$. The renormalization is due to the contribution of higher $W_{m\geq 2}$ modes which lead [13] to the decrease of the DoS in the energy range $\epsilon \leq \omega_0/L = g\omega_0\ln g$ by the relative amount of $\delta\omega_0/\omega_0 = 1/(2\ln g) \ll 1$. At higher energies, the correction decreases as $\delta\omega_0/\omega_0 \propto (g\omega_0\ln g)^2/\epsilon^2$. This correction can be found using a general approach [16], in which the perturbative treatment of the non-zero modes leads to the "induced" terms in the 0D action. It is given by the *single-cooperon* diagram which is absent in usual normal-metal problems [1,16]; from the formal point of view, the difference stems from the absence of the BB block in the parametrization (12). A usual [16] *two-cooperon* diagram leads to the "induced" term $\propto (\epsilon/\omega_0)^2(g\ln g)^{-1}$ in the effective action of the above 0D σ-model (cf. [3] where similar result is mentioned). The possibility to neglect this term determines the upper limit of energies where purely 0D description is valid:

$$\epsilon \leq \omega_{Th} = \omega_0\sqrt{g\ln g}. \quad (16)$$

To conclude, we have derived microscopically the supersymmetric field theory for the statistics of the localized electron levels inside the vortex in a moderately clean superconductor. Our supermatrix σ-model, Eq. (11) was *derived* in the main order in the quasiclassical parameter $1/\ln\Delta\tau$. Previously proposed random-matrix approach [5] is shown to be valid in the low energy range $\epsilon \leq \omega_{Th} = [(\omega_0/\tau)\ln(1/\omega_0\tau)]^{1/2}$ where zero-dimensional σ-model is applicable. Mixing between zero- and higher modes leads to the decrease of the DoS by the relative amount of $[2\ln(1/\omega_0\tau)]^{-1}$ at the energies $\epsilon \leq \tau^{-1}\ln(1/\omega_0\tau)$.

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